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MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2024**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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This document consists of **19** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use law of the logarithm of product or quotient on each side	***B1**	Allow logs to any base, as well as decimals, throughout. $\ln 8^3 + \ln 8^{-6x}$ and $\ln 4 + \ln 5^{-2x}$. Allow for $\ln \dfrac{8^3}{4}$ and $\ln 8^{6x} - \ln 5^{2x}$. $(3-6x)\ln 8$ and $\ln 4 + \ln 5^{-2x}$ gains next DB1 as well.
	Use law of logarithm of a power involving x on ONE side, e.g. $\ln 8^3 + (-)6x\ln 8$ or $(3-6x)\ln 8$ or $(9-18x)\ln 2$ or $\ln 4 - 2x\ln 5$	**DB1**	**SC** If *B0 DB0, then allow **B1** (1/4) for a correct logarithm law seen anywhere.
	Obtain a correct linear equation in x, e.g. $(3-6x)\ln 8 = (9-18x)\ln 2 = \ln 4 - 2x\ln 5$	**B1**	If in decimals, allow small errors in 2nd and 3rd dp.
	Obtain answer $x = 0.524$	**B1**	3dp required. No working scores 0/4 marks. After *B1 DB1 to correct answer with no more log working seen, then **SC B1** for $x = 0.524$. Maximum 3/4 possible.
	Alternative Method for Question 1		
	Use laws of indices to get to $a = b^{\pm 2x}$ or $c^{\pm x}$ in a correct form so now only ONE log power law required	**(B2)**	$(8^3/4)$ and $(5/8^3)^{-2x}$ or $(5^2/8^6)^{-x}$ opposite sides or $(4/8^3)$ and $(8^3/5)^{-2x}$ or $(8^6/5^2)^{-x}$ opposite sides
	Obtain a correct linear equation in x, e.g. $\ln\dfrac{8^3}{4} = 2x\ln\dfrac{8^3}{5}$	**(B1)**	$-2x\ln(5/8^3)$ or $2x\ln(8^3/5)$ or $x\ln(8^6/5^2)$ or $-x\ln(5^2/8^6)$. **SC**: If B0 then allow **B1** (1/4) for a correct term seen anywhere. If in decimals, allow small errors in 2nd and 3rd dp.
	Obtain answer $x = 0.524$	**(B1)**	3dp required. No working scores 0/4 marks. From the first line to correct answer with no log working seen, then B2 and **SC B1** for $x = 0.524$. Maximum 3/4 possible.

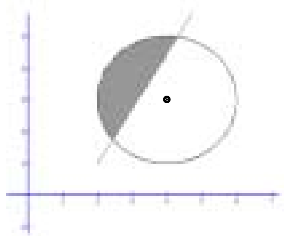
Question	Answer	Marks	Guidance
1	**Alternative Method 2 for Question 1**		
	Use laws of indices to get to any correct form with indices combined so now TWO log power laws are required	**(*B1)**	Allow 2^{7-18x} and 5^{-2x} on opposite sides or 2^{9-18x} and $2^{2-4.64x}$ on opposite sides.
	Use law of logarithm of a power involving x on ONE side, e.g. $(7-18x)\ln 2 = \ln 5^{-2x}$ or $\ln 2^{7-18x} = -2x\ln 5$ or … Allow $7-18x\ln 2$ or $9-18x\ln 2$	**(DB1)**	e.g. $(7-18x)\ln 2$ or $(9-18x)\ln 2$ or $-2x\ln 5$ or $(2-4.64x)\ln 2$ **SC:** If *B0 DB0 then allow **B1** (1/4) for a correct term seen anywhere. E.g. any term in *B1 shown above.
	Obtain a correct linear equation in x, e.g. $(7-18x)\ln 2 = -2x\ln 5$ or $(9-18x)\ln 2 = (2-4.64x)\ln 2$	**(B1)**	If in decimals, allow small errors in 2nd and 3rd dp.
	Obtain answer $x = 0.524$	**(B1)**	3dp required. No working scores 0/4 marks. From the first line to correct answer with no log working seen, then *B1 and **SC B1** for $x = 0.524$. Maximum 2/4 possible.
		4	

Question	Answer	Marks	Guidance
2	Use correct product rule $\cos 2x$ may be $1 - 2\sin^2 x$ or …	**M1**	$ae^{2x}\sin 2x + e^{2x}b\cos 2x$. Need a or $b = 2$. Allow M1 if only error is e^x instead of e^{2x} in one of terms, then maximum 1/5.
	Obtain correct derivative $2e^{2x}\sin 2x + 2e^{2x}\cos 2x$	**A1**	OE, e.g. $4e^{2x}\sin x\cos x + 2e^{2x}\left(\cos^2 x - \sin^2 x\right)$.
	Equate derivative of the form $ae^{2x}\sin 2x + e^{2x}b\cos 2x$ to 0 and solve for $2x$ or x using a correct method Note may have substituted for $\sin 2x$ and/or $\cos 2x$	**M1**	Obtain $2x = \tan^{-1}(-$ *their b/their a*$)$ OE. Allow one slip in rearranging. Allow degrees. Variety of other methods available, such as solving quadratic equation in $\sin x$ or $\tan x$ e.g. $\tan^2 x - 2\tan x - 1 = 0$ leading to $x = \tan^{-1}(1+\sqrt{2})$.
	Obtain $x = \frac{3}{8}\pi$ only or exact equivalent	**A1**	CWO 67.5° gets A0. Ignore any answers outside interval $0 \leqslant x \leqslant \frac{\pi}{2}$.
	Obtain $y = \frac{1}{2}\sqrt{2}e^{\frac{3}{4}\pi}$ only or exact simplified equivalent	**A1**	CWO, ISW. Not $\sin\left(\frac{3}{4}\pi e^{\frac{3}{4}\pi}\right)$. Ignore any answers using x outside interval $0 \leqslant x \leqslant \frac{\pi}{2}$.
		5	

Question	Answer	Marks	Guidance
3	Square $x + iy$ obtaining three terms when simplified and equate real and imaginary parts to 24 and −7 respectively	**M1**	Having used $i^2 = -1$.
	Obtain equations $x^2 - y^2 = 24$ and $2xy = -7$	**A1**	Allow $2xyi = -7i$.
	Eliminate one variable by correct method and find a horizontal equation in the other	**M1**	All powers of x or y are positive and are in the numerator.
	Obtain $4x^4 - 96x^2 - 49 = 0$ or $4y^4 + 96y^2 - 49 = 0$ or 3-term equivalents	**A1**	
	Obtain answers $\frac{7\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$ and $-\frac{7\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ or exact equivalents and no others	**A1**	E.g. $\pm\left(\frac{7\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$, but not $\pm\left(\frac{7\sqrt{2}}{2} \mp \frac{\sqrt{2}}{2}i\right)$ or $\left(\pm\frac{7\sqrt{2}}{2} \mp \frac{\sqrt{2}}{2}i\right)$. Allow coordinates or $x = \ldots$, $y = \ldots$ paired correctly. ISW converting to different form. Must simplify $\sqrt{49}$.
		5	

Question	Answer	Marks	Guidance
4	State or imply that $\ln k + \ln y = cx$ or $\ln y = cx + \ln \frac{1}{k}$ etc.	**B1**	Allow $\ln k + \ln y = cx \ln e$
	Carry out a completely correct method for finding $\ln k$ or c	**M1**	Equations must have been formulated correctly.
	Obtain value $c = 0.80$	**A1**	AWRT Allow 0.8 for 0.80. Not a fraction. Accept in the equation $ky = e^{cx}$.
	Obtain value $k = 6.5$	**A1**	AWRT Not a fraction. Accept in the equation $ky = e^{cx}$.

Question	Answer	Marks	Guidance
5	State or imply the form $A+\dfrac{B}{x-1}+\dfrac{C}{2x+1}$	**B1**	
	Use a correct method for finding a constant	**M1**	Correct appropriate method.
	Obtain one of $A = 3$, $B = 2$ and $C = –3$	**A1**	
	Obtain a second value	**A1**	
	Obtain a third value	**A1**	
	Alternative Method for Question 5		
	Divide numerator by denominator to reach $A = 3$	**(M1)**	May be implied by 3 [+] $\dfrac{ax+b}{(x-1)(2x+1)}$ with a and b not both 0.
	Obtain $3+\dfrac{x+5}{(x-1)(2x+1)}$	**(A1)**	
	State or imply the form $\dfrac{D}{x-1}+\dfrac{E}{2x+1}$	**(B1)**	
	Obtain one of $D = 2$ and $E = -3$	**(A1)**	
	Obtain a second value	**(A1)**	
		5	

Question	Answer	Marks	Guidance
6(a)	Show a circle centre (4, 3) Allow dashes for coordinates on axes	**B1**	Note full circle is not required but must show centre and include relevant arc.
	Show a circle with radius 2. Can be implied by at least two of the points (2, 3), (6, 3), (4, 1) and (4, 5) being correct	**B1FT**	FT centre not at the origin.
	Point representing (2, 1)	**B1**	Half-line or 'correct' full line extending into the third quadrant implies point (2, 1).
	Show a half-line at their (2, 1) at an angle of $\frac{1}{3}\pi$, cutting top of circle between $x = 3$ and $x = 5$	**B1FT**	FT the point (±2, ±1) or (±1, ±2).
	Shade the correct region Needs correct half-line or "correct" full line extending into the third quadrant **AND** correct circle	**B1**	
		5	
6(b)	Carry out a correct method for finding the greatest value of arg z in the correct region in (a)	**M1**	E.g. $\sin^{-1}(2/\sqrt{(25)}) + \tan^{-1}(3/4)$ or $\sin^{-1}(2/\sqrt{(25)}) + \sin^{-1}(3/5)$. Or, e.g., substitute $y = kx$ in circle equation, solve when discriminant = 0, to get $\tan^{-1}\left(\frac{6+\sqrt{21}}{6}\right)$.
	Obtain answer 1.06, or 1.05 or 1.055 or 1.056 or 60.4° or 60.5°	**A1**	The marks in **(b)** are available even if errors in **(a)**. No working seen scores 0/2 marks.
		2	

Question	Answer	Marks	Guidance
7(a)	Show $8 \times (-7)^3 + 54 \times (-7)^2 - 17 \times (-7) - 21 = 0$ This is sufficient if no errors seen. [– 2744 + 2646 +119 – 21 = 0] Or complete division of $8x^3 + 54x^2 - 17x - 21$ by $x + 7$ to get quotient $8x^2 - 2x - 3$ and remainder of 0 Or state $(x + 7)(8x^2 - 2x - 3)$ is sufficient Factors must be stated again in (b) to collect marks there	**B1**	No errors allowed. Correct division: $\begin{array}{r\|rrrr} & 8x^2 & -2x & -3 & \\ \hline x+7 & 8x^3 & +54x^2 & -17x & -21 \\ & 8x^3 & +56x^2 & & \\ \hline & & -2x^2 & -17x & \\ & & -2x^2 & -14x & \\ \hline & & & -3x & -21 \\ & & & -3x & -21 \end{array}$
		1	
7(b)	Commence division and reach partial quotient of the form $8x^2 \pm 2x$ or $8x^3 + 54x^2 - 17x - 21 = (x + 7)(Ax^2 + Bx + C)$ and reach $A = 8$ and $B = \pm 2$ or $C = -3$	**M1**	Condone no visible working.
	Obtain quotient $8x^2 - 2x - 3$ with no errors seen Stating $(x + 7)(8x^2 - 2x - 3)$ is sufficient	**A1**	Division can terminate with 0 or $-3x - 21$ stated once or twice. The working of division and finding quotient may be seen in **(a)** but results required here to collect marks.
		2	

Question	Answer	Marks	Guidance
7(c)	Solve quadratic from **(b)** to obtain a value for $\theta = \cos^{-1}\left(\frac{-1}{2}\right)$ or $\cos^{-1}\left(\frac{3}{4}\right)$	**M1**	$(x + 7)(8x^2 - 2x - 3) = (x + 7)(4x - 3)(2x + 1) = 0$ $x = \cos\theta = \frac{2 \pm \sqrt{4+96}}{16} = -\frac{1}{2}$ and $\frac{3}{4}$.
	Obtain one answer, e.g. $\theta = 120°$	**A1**	
	Obtain three further answers, e.g. $\theta = 240°$, 41.4° and 318.6° (condone 319°) and no others in the interval	**A1**	Accept more accurate answers. Answers in radians, maximum 2/3.
		3	

Question	Answer	Marks	Guidance
8(a)	State $R = \sqrt{12}$ or exact equivalent	**B1**	ISW
	Use trig formula to find α	**M1**	Allow $\alpha = 30°$ or $\tan^{-1}\left(\frac{\pm\sqrt{3}}{3}\right)$ or $\cos^{-1}\left(\pm\frac{\sqrt{3}}{2}\right)$ or $\sin^{-1}\left(\pm\frac{1}{2}\right)$ Allow M1 if $-\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ etc. NB: If $\cos\alpha = 3$ and $\sin\alpha = \sqrt{3}$ seen then M0 A0.
	Obtain $\alpha = \frac{1}{6}\pi$	**A1**	CWO, so A0 if from $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$.
		3	

Question	Answer	Marks	Guidance
8(b)	Express integral in the form $A\int \sec^2(2x+...)\,dx$ or $A\int \sec^2(2x-...)\,dx$	**B1FT**	FT α from **(a)**.
	Integrate and reach $B\tan(2x+...)$ or $B\tan(2x-...)$	**B1FT**	FT α from **(a)**. Where $B = A$ or $2A$ or $0.5A$.
	Obtain $\frac{1}{8}\tan(2x+...)$	**B1FT**	OE FT α from **(a)**. Allow $\frac{1}{8}$ as $\frac{1}{4}\times\frac{1}{2}$. Coefficient must be correct.
	Use limits of $x=0$ and $x=\frac{1}{12}\pi$ in the correct order in expression of form $B\tan(2x\pm...)$ so $B\tan\left(\frac{\pi}{6}+...\right)-B\tan(...)$ or $B\tan\left(\frac{\pi}{6}-...\right)-B\tan(-...)$	**M1**	Allow with tan still present. FT α from **(a)**. **SC:** B1 $\frac{\sqrt{3}}{12}$ OE after $\frac{1}{8}\tan\left(2x+\frac{1}{6}\pi\right)$ with no working.
	Obtain answer $\frac{1}{12}\sqrt{3}$ or $\frac{1}{4\sqrt{3}}$ or $\frac{1}{\sqrt{48}}$ or single term exact equivalent	**A1**	$\frac{1}{8}\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)=\frac{1}{8}\left(\frac{3-1}{\sqrt{3}}\right)$ needs simplifying.
		5	Note: allow all marks in **(b)** even if $\alpha=\frac{1}{6}\pi$ found by an incorrect method in **(a)**.

Question	Answer	Marks	Guidance
9(a)	Obtain $\frac{dV}{dt} = [\pm]\frac{k}{t}$ or $\frac{dV}{dt} = [\pm]\frac{1}{kt}$	**B1**	
	Obtain $\frac{dV}{dx} = 20x - 3x^2$	**B1**	
	Correct use of chain rule involving k	**M1**	Use $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$. Expressions for $\frac{dV}{dt}$ and $\frac{dV}{dx}$ must be seen to get M1.
	Obtain $\frac{dx}{dt} = [\pm]\frac{k}{t\left(20x - 3x^2\right)}$ or equivalent,	**A1**	If this expression is first seen with numerical values, allow A1 when their value of k is substituted back into the general expression.
	Use $t = \frac{1}{10}$, $x = \frac{1}{2}$ and $\frac{dx}{dt} = -\frac{20}{37}$ to obtain given answer which must be stated $\frac{dx}{dt} = -\frac{20}{37}$ needed to score final A1	**A1**	$\frac{dx}{dt} = \frac{-1}{2t\left(20x - 3x^2\right)}$ AG Need to at least see $-\frac{20}{37} = \frac{k}{\frac{1}{10}\left(10 - \frac{3}{4}\right)}$ if $\frac{k}{t}$ or $-\frac{20}{37} = \frac{-k}{\frac{1}{10}\left(10 - \frac{3}{4}\right)}$ if $-\frac{k}{t}$ in working for correct k. $\frac{dx}{dt} = \frac{20}{37}$ seen anywhere, then A0.
		5	

Question	Answer	Marks	Guidance
9(b)	Separate variables correctly & integrate at least one side correctly	**B1**	
	Obtain terms $10x^2 - x^3$	**B1**	May see $-10x^2 + x^3$ if negative sign moved across or e.g. $20x^2 - 2x^3$ if 2 moved across. Allow $\frac{20x^2}{2} - \frac{3x^3}{3}$.
	Obtain term $\ln t$ with 'correct' coefficient from their separation of variables, for example $a \ln t$ for $\frac{a}{t}$.	**B1FT**	FT sign and position of 2 from their separation but B0 if error from later manipulation.
	Use $t = \frac{1}{10}$, $x = \frac{1}{2}$ to evaluate a constant or as limits in a solution containing terms of the form x^2, x^3 and $\ln t$ (or $\ln 2t$)	**M1**	Allow numerical and sign errors and decimals. Allow if exponentiate before substitution, even if exponentiation done incorrectly, allow for c or e^c.
	Obtain correct answer in any form, for example $10x^2 - x^3 = -\frac{\ln t}{2} + \frac{19}{8} + \frac{\ln 0.1}{2}$	**A1**	$10x^2 - x^3 = -\frac{\ln 2t}{2} + \frac{19}{8} + \frac{\ln 0.2}{2}$ or $10x^2 - x^3 = -\frac{\ln t}{2} + 2.5 - 0.125 - 1.15\ldots$ Allow 1.14 to 1.16 for 1.15 and allow 2.44 to 2.46 for 2.45
	Obtain answer $t = \frac{1}{10}e^{2x^3 - 20x^2 + \frac{19}{4}}$ or equivalent	**A1**	ISW Need $t = \ldots\ldots\ldots$ E.g. $\frac{0.1}{e^{20x^2 - 2x^3 - \frac{19}{4}}}$, $\frac{e^{2x^3 + \frac{19}{4}}}{10e^{20x^2}}$, $\frac{1}{10}e^{\frac{19}{4}}e^{2x^3 - 20x^2}$. Allow decimals, allow 2.44 to 2.46 for 2.45, e.g. $e^{2x^3 - 20x^2 + 2.45}$. A0 if $e^{\ln\frac{1}{10}}$ present in final answer.
		6	

Question	Answer	Marks	Guidance
10(a)	Carry out correct process for evaluating the scalar product of direction vectors	*M1	$\left[\begin{pmatrix}3\\4\\a\end{pmatrix}.\begin{pmatrix}-1\\2\\2\end{pmatrix} \text{ or } \begin{pmatrix}3\\4\\a\end{pmatrix}.\begin{pmatrix}1\\-2\\-2\end{pmatrix}\right]$ $3(-1) + 4(2) + 2a$ or $-3 + 8 + 2a$ or $5 + 2a$. Allow one slip in unsimplified form.
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate to $\pm\frac{\sqrt{2}}{2}$, or equate the scalar product to the product of the moduli and $\pm\frac{\sqrt{2}}{2}$	*M1	*M1 marks independent of each other, so *M0 *M1 for failure to use both direction vectors, but must be using scalar product and same 2 vectors throughout. Allow $\frac{\sqrt{2}}{2}$ or $-\frac{\sqrt{2}}{2}$ throughout question.
	State a correct equation in any form, e.g. $\frac{5+2a}{3\sqrt{25+a^2}} = [\pm]\frac{\sqrt{2}}{2}$ Allow unsimplified as in guidance	A1	$\frac{5+2a}{\sqrt{9+16+a^2}\sqrt{1+4+4}} = [\pm]\frac{\sqrt{2}}{2}$ OE E.g. $5 + 2a = [\pm]\frac{\sqrt{2}}{2}\sqrt{9+16+a^2}\sqrt{1+4+4}$ If moduli initially correct but later has errors, award A1 when using $\frac{\sqrt{2}}{2}$ or $\pm\frac{\sqrt{2}}{2}$ or $-\frac{\sqrt{2}}{2}$.
	Form a quadratic equation in a with 3 or more terms all on one side and solve for a. DM1 depends on BOTH *M1	DM1	Must square $(5 + 2a)$ to get 3 terms and must remove square roots from both terms on other side. $25 + 20a + 4a^2 = \frac{9}{2}(25 + a^2)$ $a^2 - 40a + 175 = 0$ hence $(a - 5)(a - 35) = 0$.
10(a)	Obtain $a = 5$ and $a = 35$	A2	A1 for each, working not needed if quadratic correct.
		6	

Question	Answer	Marks	Guidance
10(b)	Express general point of at least one line correctly in component form, i.e. $\begin{pmatrix} 1+3\lambda \\ 1+4\lambda \\ 2a+a\lambda \end{pmatrix}$ or $\begin{pmatrix} -3-\mu \\ -1+2\mu \\ 4+2\mu \end{pmatrix}$	**B1**	Often the third point on the line occurs after M1 A1 is gained.
	Equate at least two pairs of corresponding components and solve for λ or μ or a	**M1**	If solve for a first, they must have a complete method to eliminate both λ and μ. If using a to solve for λ or for μ, a must have been found from a valid method.
	Obtain $\lambda = -1$ or $\mu = -1$	**A1**	
	Obtain $a = 2$	**A1**	
	Obtain position vector of the point of intersection is $-2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ Two different answers for point of intersection scores A0 even if one is correct	**A1**	Accept coordinates, row or column, but not $(-2\mathbf{i}, -3\mathbf{j}, +2\mathbf{k})$ or $\begin{pmatrix} -2\mathbf{i} \\ -3\mathbf{j} \\ 2\mathbf{k} \end{pmatrix}$ but ISW after correct form seen.
		5	